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AERODYNAMICS OF LAMINAR DIFFUSIVE FLAMES

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AERODYNAMICS OF LAMINAR DIFFUSIVE FLAMES

L. P. Yarin

It is characteristic for intensive combustion of unmixed gasses that there is a clear leading edge of the flame, dividing the flow field into two areas, one of which contains the fuel and combustion products, while the other contains the oxidizer and combustion products. The structure of the flame corresponds to complete mixing of the components and infinite chemical reaction rate at the leading edge of the flame. The pressure of only one of the two reagents in each of the two areas allows the Burke-Schuman-Zel'dovich model [1, 2] to be used effectively for determination of the primary characteristics of the gas flame [3, 4]. This article studies the aerodynamics of a number of characteristic types of straight-stream laminar gas flames in the framework of this model, including a free flame, a semilimited flame and a flame propagating in a wake. The solution is presented within the framework of boundary layer theory, based on a generalized plan for calculation of a diffusion flame [4]. /395*

1. Let us study the aerodynamics of a flat laminar flame, formed as a gas stream leaves a narrow rather long slit. We will assume that the pressure is constant throughout the flow area, $\mu\rho$=const, that the Lewis number is equal to one. This allows us

*Numbers in the margin indicate pagination in the foreign text.

to limit ourselves to solving only the dynamic and diffusion problems. We will also ignore changes in molecular weight during the process of the reaction.

Under these assumptions, the problem is reduced to integration of the following system of equations:

$$\rho u \frac{\partial u}{\partial x} + \rho V \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \quad \frac{\partial \rho u}{\partial x} + \frac{\partial \rho V}{\partial y} = 0,$$
$$\rho u \frac{\partial \widetilde{\Delta c}}{\partial x} + \rho V \frac{\partial \widetilde{\Delta c}}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial \widetilde{\Delta c}}{\partial y}\right); \tag{1}$$

with the boundary conditions: /396

$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial \widetilde{\Delta c}}{\partial y} = 0$ where $y = 0$; $u = 0, \quad \widetilde{\Delta c} = 0$ where $y \to \infty$

where $\widetilde{\Delta c} = \bar{c}_1 - \bar{c}_2 + 1, \quad \bar{c}_1 = \frac{c_1}{c_{2,\infty}}\Omega, \quad \bar{c}_2 = \frac{c_2}{c_{2,\infty}}$ are the concentration of fuel and oxidizer respectively; Ω is the stoichiometric coefficient.

Let us also write the integral relations replacing the initial conditions:

$$\int_0^\infty \rho u^2 dy = I_0; \quad \int_0^\infty \rho u \widetilde{\Delta c}\, dy = G_0 \quad (I_0,\ G_0 - \text{const}).$$

Making a transition to the plane of variables $\xi = x, \ \eta = \int_0^y \rho dy$, we will seek the self-similar solution to equation system (1) as:

$$\frac{u}{u_m} = F'(\psi), \quad \frac{\widetilde{\Delta c}}{\widetilde{\Delta c}_m} = \pi(\psi), \quad u_m = A\xi^\alpha, \quad \widetilde{\Delta c} = \Gamma\xi^\gamma, \quad \psi = B\eta\xi^\beta$$

(A, B, Γ are constants, defined by the integral characteristics of the stream I_0, G_0; $\alpha=\gamma=\beta/2=-1/3$). The expressions defining the values of the desired functions are [5]:

$$F'(\psi) = (\mathrm{Ch}\,\psi)^{-2}, \quad \pi(\psi) = (\mathrm{Ch}\,\psi)^{-2Pr}. \tag{2}$$

Considering (2), we can write, following [4], relationships defining the distribution of concentration in the transverse cross sections of the flame:

$$\frac{c_1}{c_{1,m}} = \frac{(\mathrm{Ch}\,\psi_\phi)^{2Pr} - (\mathrm{Ch}\,\psi)^{2Pr}}{(\mathrm{Ch}\,\psi)^{2Pr}\left[(\mathrm{Ch}\,\psi_\phi)^{2Pr} - 1\right]} \tag{3}$$

for the internal area of the flame, and

$$\frac{c_2}{c_{2,}} = 1 - \left(\frac{\mathrm{Ch}\,\psi_\phi}{\mathrm{Ch}\,\psi}\right)^{2Pr} \tag{4}$$

for the external area[1].

Introducing the flame length l_ϕ as the characteristic scale, let us write an equation defining the change in fuel concentration along the axis of the flame, and an equation relating the coordinates of the leading edge of the flame:

$$\bar{c}_{1,m} \quad (1-\bar{\xi}^{1/3})\,\bar{\xi}^{1/3}, \tag{5}$$

$$\bar{\xi}^{1/3} \quad (\mathrm{Ch}\,\psi_\phi)^{2Pr} - 1, \tag{6}$$

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[1] Here and below, the subscript "ϕ" indicates the välue of a quantity at the leading edge of the flame.

where $\bar{\xi} = \xi l_{\phi}$; $\bar{c}_{1,m} = \frac{c_{1,m}}{c_{1,0}}$; $c_{1,0}$ is the scale of concentration.

If we make a transition from the stream of the source to a stream of finite size having the same integral characteristics, and assume that the initial concentration of fuel at the output /397 of the effective nozzle is $c_{1.0}$, then as the fuel flows out without inert impurities ($c_{1.0}=1$), calculation of $\bar{c}_{1.m}$ using equation (5) can be performed only in the range of change of $\bar{\xi}$ of from 0.152 to 1. This limitation reflects one peculiarity of the self-similar solution, in that it does not allow calculations in the area of the nozzle.

The change in temperature along the axis of the flame can be found from the condition of similarity of the profiles of total enthalpy and the generalized concentration

$$\frac{\Delta I_m}{\Delta I_0} = \frac{\widetilde{\Delta c}_m}{\widetilde{\Delta c}_0}, \quad (I = i + qc),$$

from which:

$$i_m - i_\infty = \frac{q_1 c_{1,0}}{1+\beta}\left(\frac{c_{1,m}}{c_2}\Omega + 1\right) - q_1 c_{1,m}, \tag{7}$$

where q is the efficiency of the fuel; $\beta = \frac{c_{1,0}}{c_{2,\infty}}\Omega$. Using (3) and (6) and considering that $i_\phi - i_\infty = \frac{q_1 c_{1,0}}{1+\beta}$, let us transform equation (7) to

$$i_m - i_\infty = (i_\phi - i_\infty)\left[2 - (\mathrm{Ch}\,\psi_\phi)^{2\mathrm{Pr}}\right]. \tag{8}$$

Relationships (2)-(8) allow us to determine the aerodynamic characteristics of the flame once we have found its length.

This is determined from the condition of equality of the potential chemical energy of the source to the summary heat flux passing through the transverse cross section of the flame, corresponding to the value of the coordinate $x=l_{\phi}$ [6].

Figure 1 shows the change in velocity, temperature and concentration along the axis of the flame. The curves are universal in nature, since they relate to various flow conditions--various initial values of fuel concentration, oxidizer concentration in the surrounding space, Re number, etc. This generalization is possible due to the selection of the flame length as the characteristic scale. Obviously, with this normalization the parameters influence the flame length, which acts as the main scale characteristic of the diffusion flame.

The distribution of characteristic quantities in the field of flow of the flat laminar flame is shown on Figures 2 and 3[2]. They show the fields of velocities and temperatures, flux density and momentum flow density, and also (see Figure 2) the flow lines and flame leading edge lines. The graphs show that within the framework of the calculation plan used there is a discontinuity of derivatives of momentum flow density across the leading edge of the flame. However, as calculations have shown, the change of $(\rho u^2)'$ at the leading edge of a diffusion flame is slight, which

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[2] $\phi=By?x^3$

allows us to assume profiles ρu^2 smooth in approximate calculations. This is broadly used in calculating turbulent diffusion flames [3].

The influence of a number of parameters on the configuration of the flame is illustrated by Figure 4. We can see from this graph that a change in the initial concentration of the fuel causes a shortening of the flame and a decrease in its width[3]. The value of the Pr number and exhaust velocity of the gas also have a significant influence.

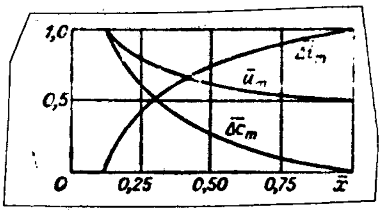

Figure 1. Change in velocity, temperature and concentration of fuel along flame axis

2. A semibounded flame. As we know, the aerodynamics of a semibounded flame have inherent peculiarities, characteristic both for the free boundary layer and for a boundary layer next to a solid surface. It should be noted that, depending on the type of boundary conditions at the wall $\left(\lambda \frac{\partial T}{\partial y} = 0,\ \lambda \frac{\partial T}{\partial y} = f(x),\ T_w = \mathrm{const}\right)$, both the calculation plan and the very statement of the problem of combustion of the flame change. /398

For example, in particular, the condition T_w=const cannot be combined with the basic assumption of diffusion flame theory, that of constancy of temperature across the leading edge of the flame. Actually, at the tip of the flame, i.e., at the point where the leading edge contacts the surface of the solid, two conditions

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[3] For convenience in comparison of data with various values, the longitudinal coordinates are related to the flame length at $c_{1.0}$=?, the final coordinates are calculated to y_{max} at $c_{1.0}$=1.

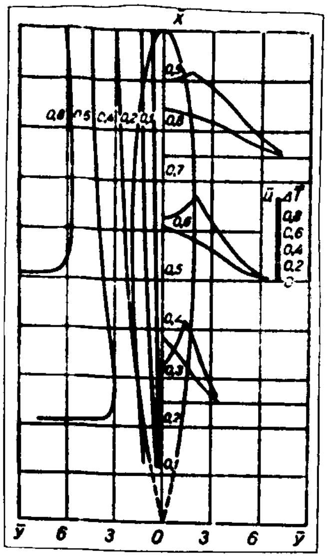

Figure 2. Velocity and Temperature Distribution in Flat Laminar Flame ($c_{1.0}$=1, $c_{2.\infty}$= =0.23, Pr=1, T_ϕ/T_∞=7.17)

should be fulfilled simultaneously: $T=T_W$ and $T=T_\phi$. With arbitrary fixation of wall temperature, these conditions are obviously incompatible. Therefore, the solution of the problem of combustion of a semibound flame where T_W=const can be produced only by considering the kinetics of the process, when the change in temperature along the leading edge is determined by the thermal mode of combustion. We note that with intensive heat transfer and correspondingly sharp supercooling of the reaction zone, combustion at the tip of the flame may be converted from the diffusion area to the kinetic area. As a result, there will be no clearly expressed leading edge to the flame in this area.

Without studying the general statement of the problem, let us limit ourselves to discussion of the results relating to the particular case of combustion of a gas near an adiabatic wall. In solving the problem of diffusion combustion with a jet flowing around a nonconductive plate, we must integrate equation system (1) considering the following boundary and integral conditions:

$$u=0,\ \frac{\partial \widetilde{\Delta c}}{\partial y}=0 \quad \text{where} \quad y=0;\ u=0,\ \widetilde{\Delta c}=0 \quad \text{where} \quad y \cdots;$$

$$\int_0^\infty \rho u^2 \left(\int_0^y \rho u\, dy\right) d y = \text{const}, \quad \int_0^\infty \rho u \widetilde{\Delta c}\, d y = \text{const}.$$

Assuming $u/u_m = F(\psi)$, $u_m = A\xi^{\alpha}$, etc., we can write the solution to the equations of motion and diffusion as [5, 7]:

$$\psi = \frac{1}{2F_\infty} \ln \frac{F + \sqrt{F \cdot F_\infty} + F_\infty}{(\sqrt{F} - \sqrt{F_\infty})^2} + \frac{\sqrt{3}}{F_\infty} \left[\arctan \frac{2\sqrt{F} + \sqrt{F_\infty}}{\sqrt{3F_\infty}} - \arctan \frac{1}{\sqrt{3}}\right] \tag{9}$$

$$\pi(\psi) = \exp\left(-\Pr \int_0^{\psi} F\, d\psi\right). \tag{10}$$

Using these relationships, let us write expressions describing the /399 distribution of concentrations of reagents and temperature in transverse cross sections of the flame:

$$\frac{c_1}{c_{1,m}} = \frac{i - i_\Phi}{i_m - i_\Phi} = \frac{\exp\left(-\Pr \int_0^{\psi} F\, d\psi\right) - \exp\left(-\Pr \int_0^{\psi_\Phi} F\, d\psi\right)}{1 - \exp\left(-\Pr \int_0^{\psi_\Phi} F\, d\psi\right)}, \tag{11}$$

$$\bar{c}_2 = \frac{i_\Phi - i}{i_\Phi - i_\infty} = 1 - \exp\left[\Pr\left(\int_0^{\psi_\Phi} F\, d\psi - \int_0^{\psi} F\, d\psi\right)\right] \tag{12}$$

correspondingly for the internal and external areas of the flame. The equation relating the coordinates of the leading edge of the flame is:

$$\exp\left(-\Pr \int_0^{\psi_\Phi} F\, d\psi\right) = \xi^{1/3}. \tag{13}$$

Figure 5 shows the distribution of characteristic quantities in the flow field of a semibounded flame. We can see from the graph that with increasing distance from the mouth, the temperature field is gradually smoothed. Significant smoothing of the temperature field in the internal area of the flame results from the increase in wall temperature with increasing distance from the source. As concerns the distribution of velocities in the transverse cross sections of the flame, it is similar to the distribution of velocity in semibounded streams without combustion. In a flame, as in a stream, the longitudinal component of the velocity vector has a maximum located at some distance from the wall. The displacement of the maximum of the velocity (for fixed values of ξ) depends on the heat liberated in the combustion zone--the heat content of the fuel. This relationship appears implicitly as the dependence of transverse coordinate y on density field and is detected upon transition from the plane of variables ξ and η to the plane of variables x and y.

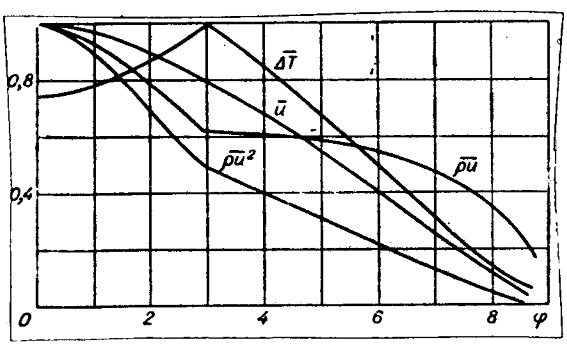

Figure 3. Distribution of Velocity, Flux Density, Momentum Flow Density and Temperature in the Transverse Cross Section of a Flat Laminar Flame ($\bar{x}$=0.5; $c_{1.0}$=1, c_∞=0.23, T_ϕ/T_∞= =7.17)

3. A flame in a wake. The commonest type of straight-stream flame is the flame propagating in a wake. It has both the characteristic properties of flooded gas flames, and certain specific peculiarities resulting from the aerodynamics of wake flow. In particular, in a wake flame the distribution of velocity, temperature and concentration, length and shape of

the flame depend not only on the initial values of u, T, c and the physical and chemical properties of the reagents, but also on the relationship between the velocity of the stream and the velocity of the flow. One significant peculiarity (from the standpoint of construction of calculation) of a wake flame is the non- /400 self-similarity of the flow. Therefore, we shall use integral methods of calculation of free streams to analyze the aerodynamics of a flame propagating in a wake [8].

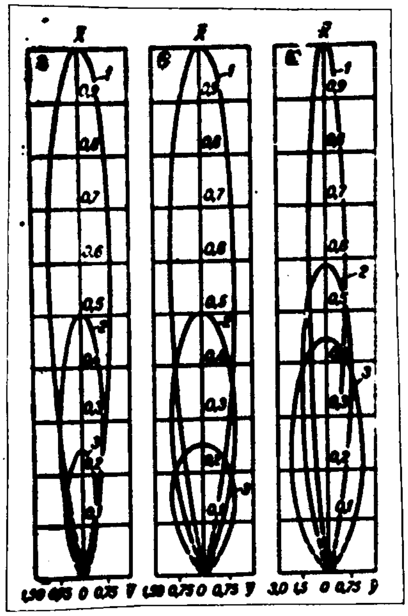

Figure 4. Configuration of a Flat Laminar Flame.

а) 1 — $\bar{c}=1$; 2 — $\bar{c}=0.8$, 3 — $\bar{c}=0.6$; Re — const, Pr=1; б) 1 — $\overline{Re}=1$, 2 — $\overline{Re}=0.5$, 3 — $\overline{Re}=0.25$; $c_{1,0}$ — const, Pr=1; в) 1 — Pr=1, 2 — Pr=0.5, 3 — Pr=0.25, $c_{1,0}$ — const, Re — const.

Let us write the equations of motion and diffusion in Dorodnitsyn variables, after combining them to the continuity equation:

$$\boxed{\frac{\partial u(u-u_\infty)}{\partial \xi} + \frac{\partial V(u-u_\infty)}{\partial \eta} = \nu_\infty \frac{\partial^2 (u-u_\infty)}{\partial \eta^2},} \tag{14}$$

$$\boxed{\frac{\partial u\widetilde{\Delta c}}{\partial \xi} + \frac{\partial V\widetilde{\Delta c}}{\partial \eta} = D_\infty \frac{\partial^2 \widetilde{\Delta c}}{\partial \eta^2},} \tag{15}$$

and represent the desired functions $u\Delta u$ and $u\widetilde{\Delta c}$ as:

$$u(u-u_\infty) = \sum_0^n a_n y^n, \quad u\widetilde{\Delta c} = \sum_0^n b_n y^n,$$

where u_∞ is the velocity of the wake.

Limiting ourselves to third power polynomials, let us determine the value of the coefficients from the additions on the axis and the external boundary of the boundary layer:

$$\frac{\partial u(u-u_\infty)}{\partial \eta}=0, \quad u_m \frac{d(u_m-u_\infty)}{d\xi}=\nu_\infty \frac{\partial^2(u-u_\infty)}{\partial \eta^2}\Big|_{\eta=0},$$

$$\frac{\partial u \widetilde{\Delta c}}{\partial \eta}=0 \quad \text{where} \quad \eta=0,$$

$$u(u-u_\infty)=0, \quad u\widetilde{\Delta c}=0, \quad \frac{\partial u(u-u_\infty)}{\partial \eta}=0, \quad \frac{\partial u \widetilde{\Delta c}}{\partial \eta}=0 \quad \text{where} \quad \eta=\delta,$$

where δ is the thickness of the boundary layer.

As a result, we produce:

$$u(u-u_\infty)=-\frac{(2u_m-u_\infty)u_m}{\sigma\nu_\infty}\frac{d(u_m-u_\infty)}{d\xi}\delta^2 F(\bar{\eta}), \tag{16}$$

$$u\widetilde{\Delta c}=u_m\widetilde{\Delta c}_m F(\bar{\eta}), \tag{17}$$

$$F(\bar{\eta})=1-3\bar{\eta}^2+2\bar{\eta}^3, \quad \bar{\eta}=\frac{\eta}{\delta}.$$

where

Assuming in (16) that $\bar{\eta}=0$, we produce an equation relating the value of velocity on the axis to the thickness of the boundary layer:

$$u_m-u_\infty=-\frac{2u_m-u_\infty}{\sigma\nu_\infty}\frac{d(u_m-u_\infty)}{d\xi}\delta^2. \tag{18}$$

Based on the condition of retention of excess momentum /401
$2\int_0^\delta \rho u(u-u_\infty)\,dy=I_0$, we have:

$$u_m(u_m-u_\infty)=\frac{I_0}{2\delta\rho\int_0^1 F(\bar{\eta})\,d\bar{\eta}}. \tag{19}$$

Considering this relationship, equation (18) can be written as:

$$d\bar{\xi}=\frac{R_0}{12}\left\{\sqrt{\bar{\delta}}\sqrt{m^2\bar{\delta}+4(1-m)}+m\bar{\delta}\right\}d\bar{\delta}, \tag{20}$$

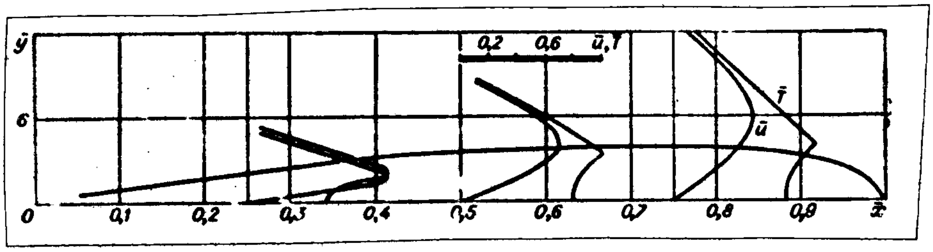

Figure 5. Distribution of Velocity and Temperature in Semibounded Flame ($c_{1.0}=1$, $c_{2.\infty}=0.23$, $T_\phi/T_\infty=7.17$)

where $m=\frac{u_\infty}{u_0}$, $\bar{\delta}=\frac{\delta}{L_0}$, $\bar{\xi}=\frac{\xi}{L_0}$, $R_0=\frac{u_0 L_0}{\nu_0}$, L_0 is the height of the nozzle. Integrating equation (20) from $\bar{\xi}_0$ to $\bar{\xi}$ and 1 to $\bar{\delta}$, we produce

$$\bar{\xi}-\bar{\xi}_0=\frac{R_0}{12}\left\{\frac{1}{2m^3}\left(m\sqrt{\bar{\delta}}\,[m^2\bar{\delta}+4(1-m)]^{1.5}-2(1-m)\times\right.\right.$$
$$\left.\times\left[m\sqrt{\bar{\delta}}\sqrt{m^2\bar{\delta}+4(1-m)}+4(1-m)\ln\left(m\sqrt{\bar{\delta}}+\sqrt{m^2\bar{\delta}+4(1-m)}\right]\right)+$$
$$\left.+\frac{m^2\bar{\delta}^2}{2}\right\}-z(m), \tag{21}$$

where $z(m)=\frac{R_0}{12}\left\{\frac{1}{2m^3}\left(m\,[m^2+4(1-m)]^{1.5}-2(1-m)\left[m\sqrt{m^2+4(1-m)}\right.\right.\right.+$

$\left.\left.\left.+4(1-m)\ln(m+\sqrt{m^2+4(1-m)}\right]\right)+\frac{m}{2}\right\}$ is the length of the initial sector. In a wake flow, the length of the initial sector is a function of parameter m. Therefore, in calculating total length of the flame, we must consider the function $\xi_0(m)$. To determine it, we write the integral conditions as follows (Figure 6):

$$\int_0^{\eta_\delta} u(u-u_x)\,d\eta=\int_0^{\eta_0} u_0(u_0-u_x)\,d\eta+\int_0^{\eta_\delta} u(u-u_x)\,d\eta, \tag{22}$$

$$\left|\int_0^{\eta_1} u_0(u_0^2-u_x^2)\,d\eta-\int_{\eta_1}^{\eta_\delta} u(u^2-u_x^2)\,d\eta\right|=-2\int_0^{\infty}\nu_\tau\left(\frac{\partial u}{\partial\eta}\right)^2 d\eta. \tag{23}$$

Assuming $u(u-u_\infty)/u_0(u_0-u_\infty)=F(\bar\eta)$, after several transforms we produce

$$\boxed{\bar\eta_0 + (\bar\eta_\delta - \bar\eta_0)\int_0^1 F(\bar\eta)\,d\eta = 0{,}5,} \qquad (24)$$ /402

$$\boxed{(1+m)\frac{d\bar\eta_0}{d\bar\xi} + \Phi_1\frac{d(\bar\eta_\delta-\bar\eta_0)}{d\bar\xi} = -\frac{1}{2R_0}\Phi_2\frac{1-m}{\bar\eta_\delta-\bar\eta_0},} \qquad (25)$$

where

$$\Phi_1 = 0{,}5\int_0^1 F(\bar\eta)\left[\sqrt{m^2+4(1-m)F(\bar\eta)}+3m\right]d\eta,$$

$$\Phi_2 = \int_0^1 \frac{[F'(\bar\eta)]}{m^2+4(1-m)F(\bar\eta)}\,d\bar\eta, \quad \bar\eta = \frac{\eta-\eta_0}{\eta_\delta-\eta_0}.$$

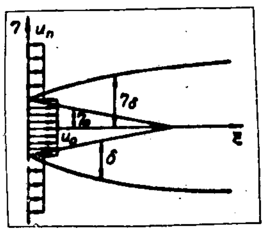

Figure 6. Diagram of Flow in Initial Sector of Wake Flame

Let us determine from (24) the derivative $d\bar\eta_0/d\xi$ and substitute its value into (25). As a result, we produce:

$$\boxed{(\bar\eta_\delta-\bar\eta_0)\frac{d(\bar\eta_\delta-\bar\eta_0)}{d\bar\xi} = \frac{1}{2R_0}(1-m)\frac{\Phi_2}{(1-m)-\Phi_1}.}$$

Let us integrate this equation considering $\bar\delta_0=0$, $\bar\eta_\delta=1$ at the end of the initial sector:

$$\boxed{\bar\xi_0 = \frac{R_0}{1-m}\frac{(1+m)-\Phi_1}{\Phi_2}.} \qquad (26)$$

Equation system (19), (21), (26) allows us to determine the velocity on the axis of the flame. Changes in fuel concentration on the axis can be found from the condition of conservation of flow of generalized concentration:

$$\tilde{\Delta c} = \frac{G_0}{u_m \delta \rho_\infty \int\limits_0^1 F(\bar{\eta})\, d\bar{\eta}}.$$

Considering that where $\bar{\xi}=\bar{l}_\phi$, $c_{1,m}=0$, we produce:

$$\bar{c}_{1,m} = \frac{u_m(\bar{l}_\phi)\,\delta(\bar{l}_\phi)}{u_m(\bar{\xi})\,\delta(\bar{\xi})} - 1, \qquad (27)$$

where $u_m(\bar{l}_\phi)$ and $\delta(\bar{l}_\phi)$ are the velocity and thickness of the boundary layer at the end of the flame respectively. Using (19), we can write equation (27) as:

$$\bar{c}_{1,m} = \frac{\delta(\bar{l}_\phi)\left[m + \sqrt{m^2 + \frac{4(1-m)}{\delta(\bar{l}_\phi)}}\right]}{\delta(\bar{\xi})\left[m + \sqrt{m^2 + \frac{4(1-m)}{\delta(\bar{\xi})}}\right]}. \qquad (28)$$

The distribution of concentration in the transverse cross sections of the flame is:

$$\frac{c_1}{c_{1,m}} = \frac{F(\bar{\eta})\frac{u_m}{u} - F(\bar{\eta}_\phi)\frac{u_m}{u_\phi}}{1 - F(\bar{\eta}_\phi)}, \qquad (29)$$

$$\bar{c}_2 = 1 - \frac{F(\bar{\eta})}{F(\bar{\eta}_\phi)}\,\frac{u_\phi}{u} \qquad (30)$$ /403

respectively for the internal and external areas of the flame. In conclusion, let us define the equation relating the coordinates of the leading edge of the flame and the dependence of flame length on fixed parameters. Assuming in (17) that $\bar{\eta}=\bar{\eta}_\phi(\bar{c}_1=0)$ and considering that $u(\bar{\eta}_\phi)=u(\bar{l}_\phi)$, we produce:

$$\boxed{F(\bar{\eta}_\Phi)\frac{\delta(\bar{l}_\Phi)}{\delta(\bar{\xi}_\Phi)}=1.} \qquad (31)$$

The equality of the potential chemical energy of the source of summary heat flux passing through the transverse cross section of the flame and the corresponding end indicates that[4]

$$\boxed{u(\bar{l}_\Phi)\,\delta(\bar{l}_\Phi)=\beta+1.} \qquad (32)$$

Using (19), we can transform equation (32) to:

$$\boxed{\delta(\bar{l}_\Phi)=\frac{(1+\beta)^2}{1+\beta m}.} \qquad (33)$$

Simultaneous solution of equations (21), (22) and (33) allows us to determine the function $\bar{l}_\phi=f(\beta, m)$. This solution shows that the distribution of velocity, temperature and concentration in a wake depends significantly on the relationship of velocities of the gas stream and the wake. The value of this ratio, which goes far toward determining the intensity of the mixing process, has a significant influence on the configuration of the leading edge of the flame and the length of the flame. An increase of parameter m causes (in the area $m<1$) a great increase in flame length. With

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[4] In writing the equations for heat balance, it was assumed that the summary heat production in the flame qG_0 is significantly greater than the initial flux of heat content $qG_0 \gg \rho_0 u_0 \Delta i_0 L_0$.

wake velocities greater than the initial flow velocity of the stream ($m>1$), l_ϕ decreases with increasing m.

Laminar diffusion flames of various types characteristically show a linear dependence of flame length on Reynolds number. This number has a significant influence on the configuration of the flame, distribution of velocity and temperature and concentration in the flow field. One of the most significant factors determining the aerodynamics of a flame is the relationship of concentrations of reagents and their stoichiometric ratio. It is remarkable that for various types of diffusion flames, the influence of concentration of the components and Ω on the basic characteristics of the flame is reflected by the single stoichiometric complex $\beta=(c_{1.0}/c_{2.\infty})\Omega$. A change in β (for example, ballasting of the fuel with an inert gas) causes a sharp change in flame dimensions.

We note also that the statement used can be applied to the study of the aerodynamics of combustion of a gas in other types of stream flows allowing self-similar solution of the dynamic problem. As in the examples above, the analytic solution can be found only for particular cases of the dependence of transfer coefficients on temperature. This limitation is not too rigid for qualitative study of the influence of basic parameters on the characteristics of gas flames. The solution of the problem in its /404
total volume considering the complex functions $\mu(T)$ and $\lambda(T)$, etc. can be found apparently only by numerical solution using computers.

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